ASTR 421

Stellar Observations and Theory

Lecture 12

Stellar Structure

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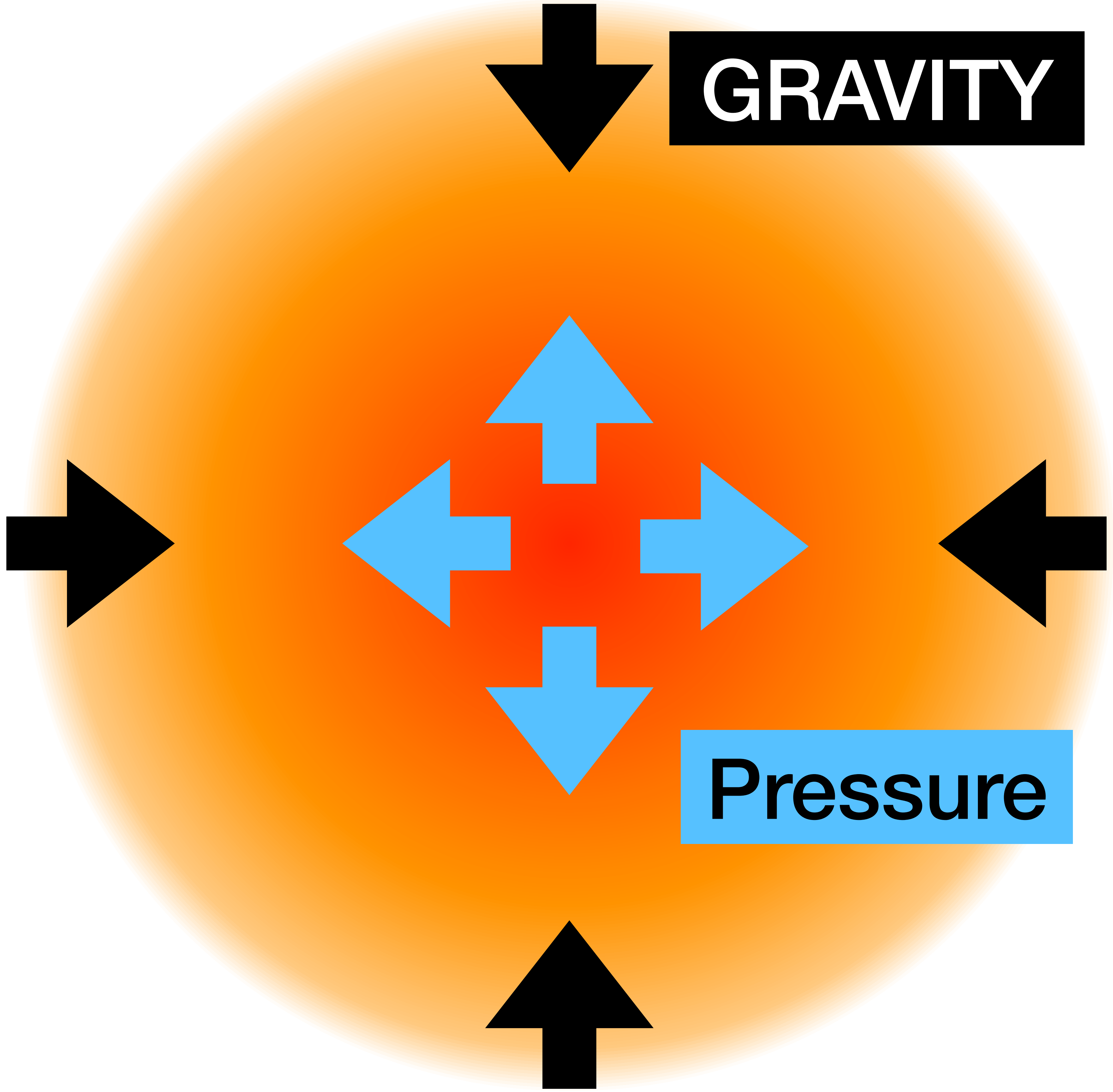
Today

- Stellar structure
- Boundary conditions of a star
- General overview of interior structure of the Sun
- Timescales for stars

Hydrostatic Equilibrium

- Star must stay in balance between its own self-gravity, and pressure support
- This is both a simple/obvious statement (the star would otherwise destroy itself) and a profound/critical condition for understanding stellar structure!
- Pressure support initially comes from fusion in core, gas pressure ("equation of state") supports throughout star



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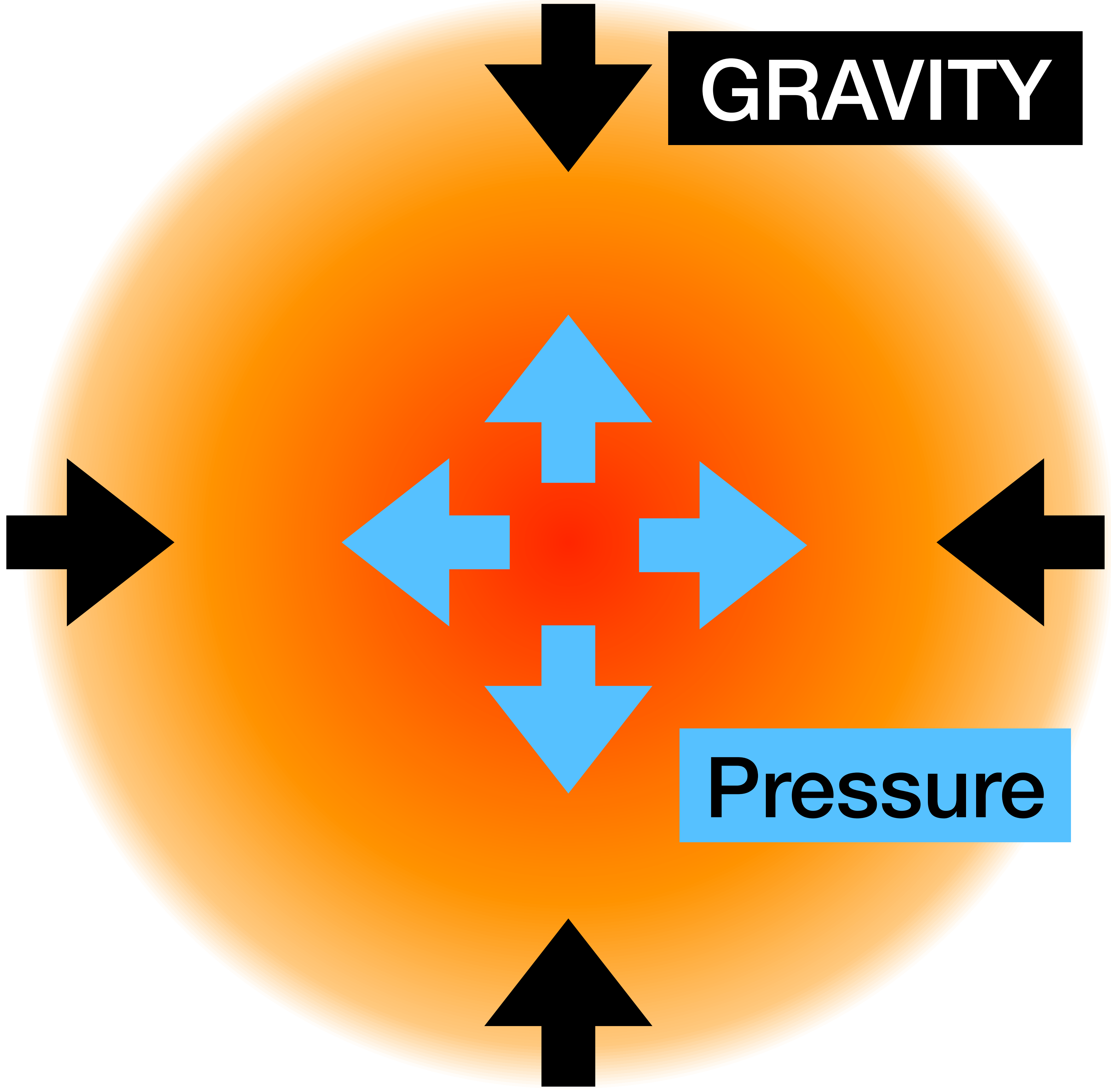
Hydrostatic Equilibrium

- It must be self-governing…
  e.g. if fusion were to *increase*
  - Temperature increases
  - Pressure increases
  - Core expands
  - Temperature & density drops, fusion rate drops!
- & visa-versa… the constant downward pressure from gravity ensures the star will keep fusion going in the core! (as long as it *has fuel…*)


GRAVITY

Pressure

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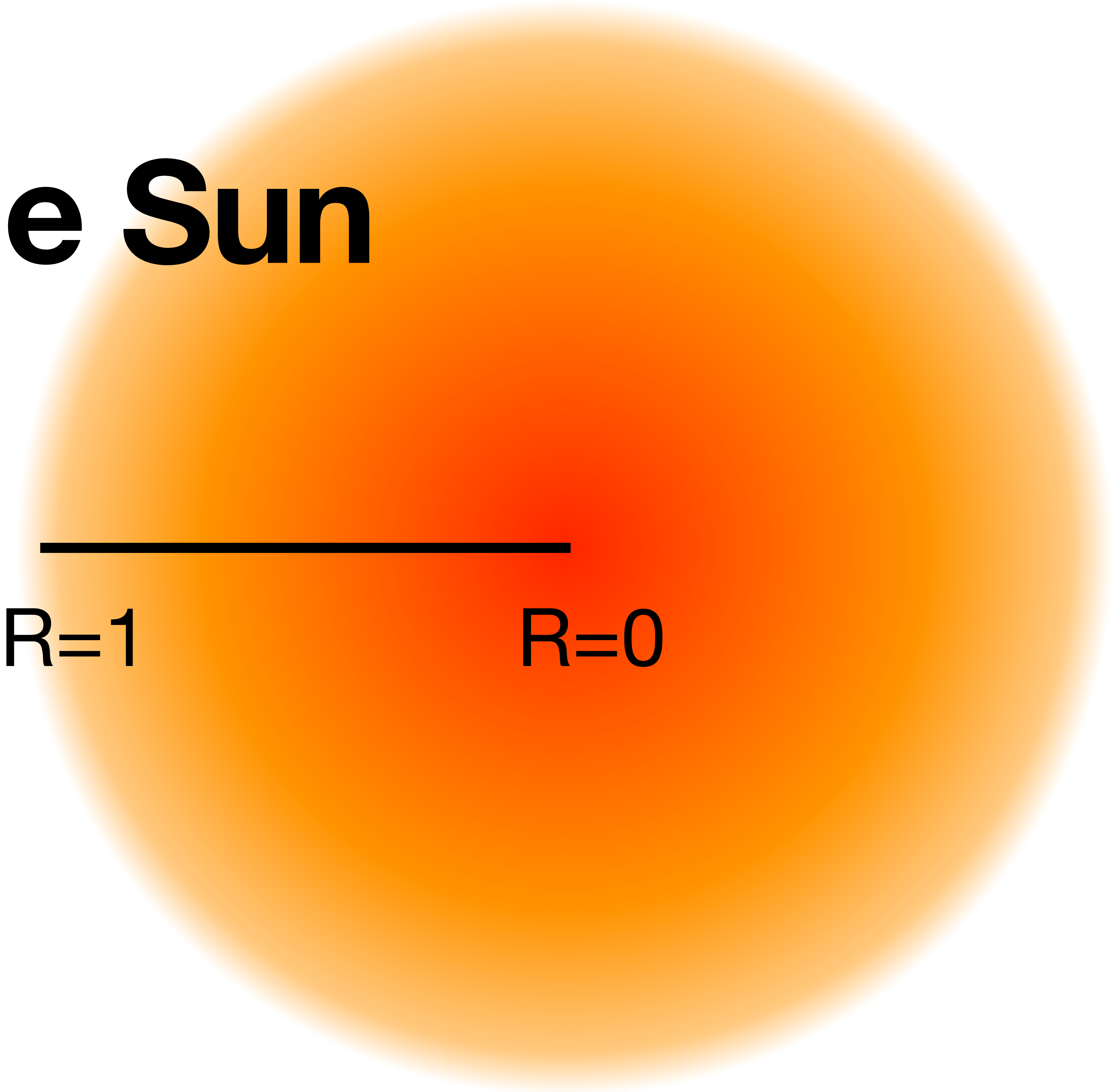
Boundary Conditions for the Sun

- **At R=0:** M=0, P=Pc, T=Tc
- **At R=1:** M=1, P=0, T=0
- $L_\odot = 4 \times 10^{33}$ erg / s
- $M_\odot = 2 \times 10^{33}$ g
- $R_\odot = 7 \times 10^{10}$ cm
- H: 73% of mass (X)
  He: 26% of mass (Y)
  metals: ~1% of mass (Z)



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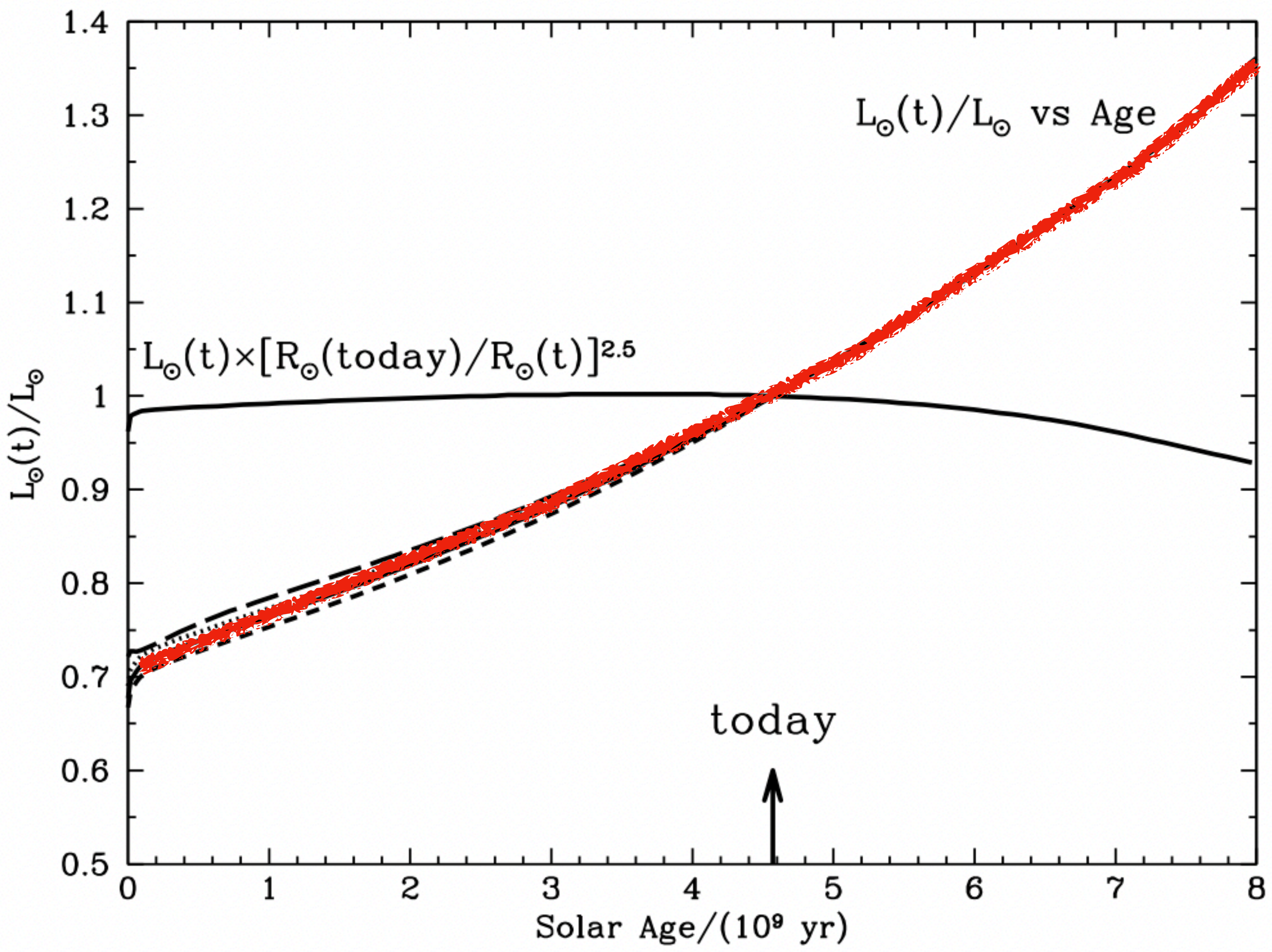
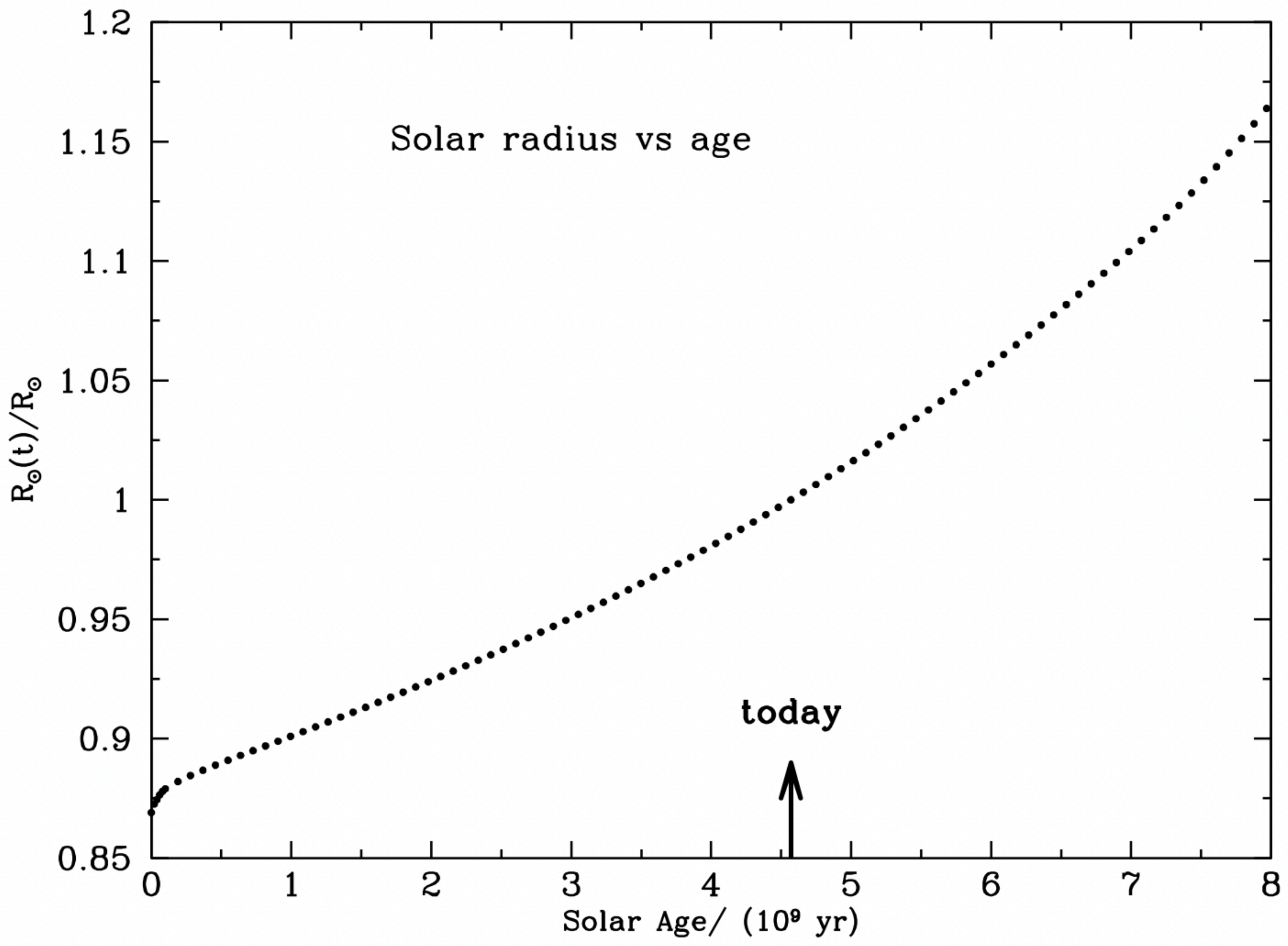
Standard Solar Model

- “BP2000”
  Bachall, Pinsonneault, & Basu (2001)
- Widely used reference for interior structure of the Sun
- Includes model for *changes* in the structure of Sun over its lifetime!





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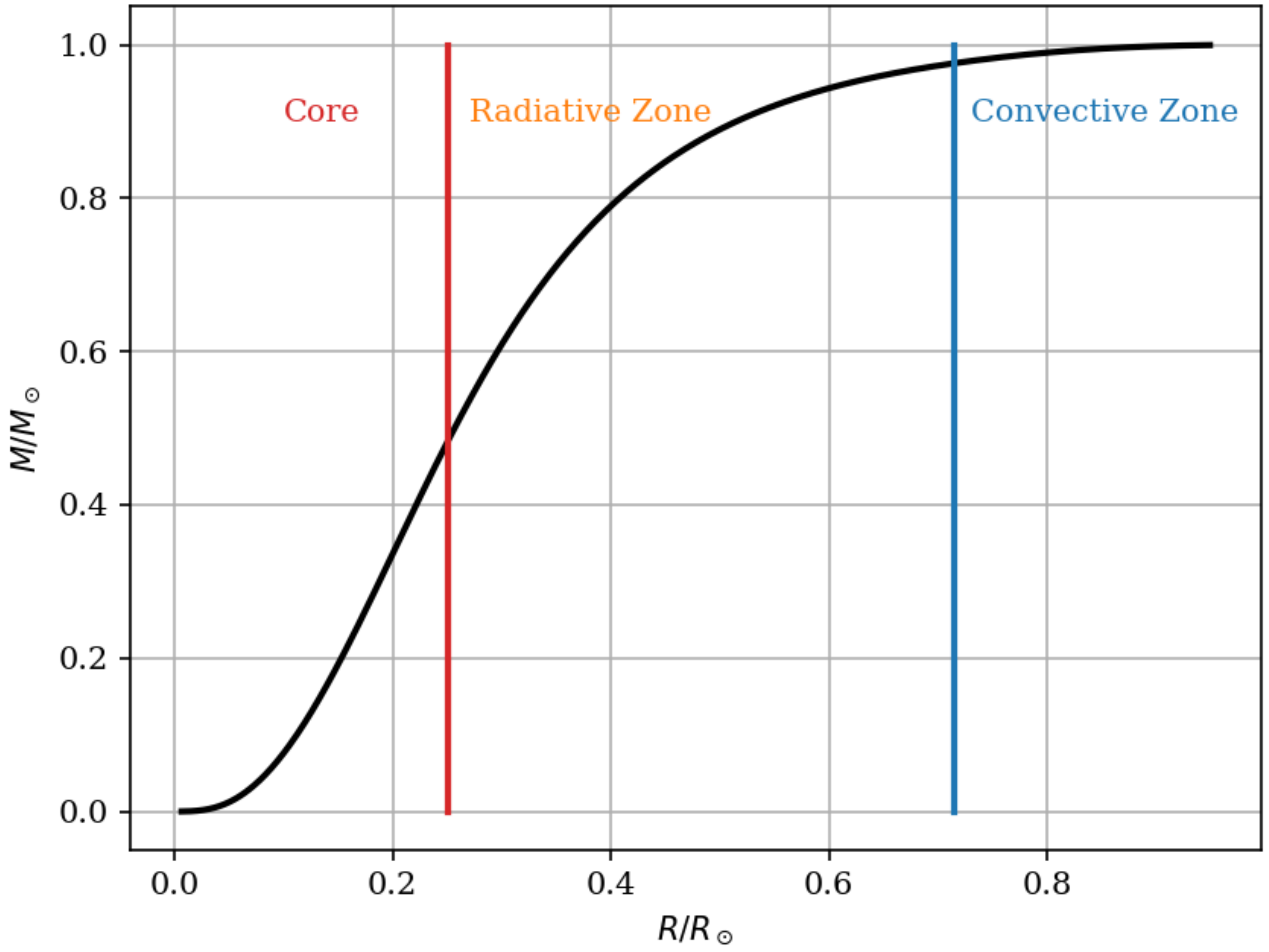
Standard Solar Model

- Almost 90% of the mass is contained in the inner 50% of radius
    - Very dense inside!
- Core reaches ~0.25 $R_\odot$
- Convective zone starts at ~0.714 $R_\odot$



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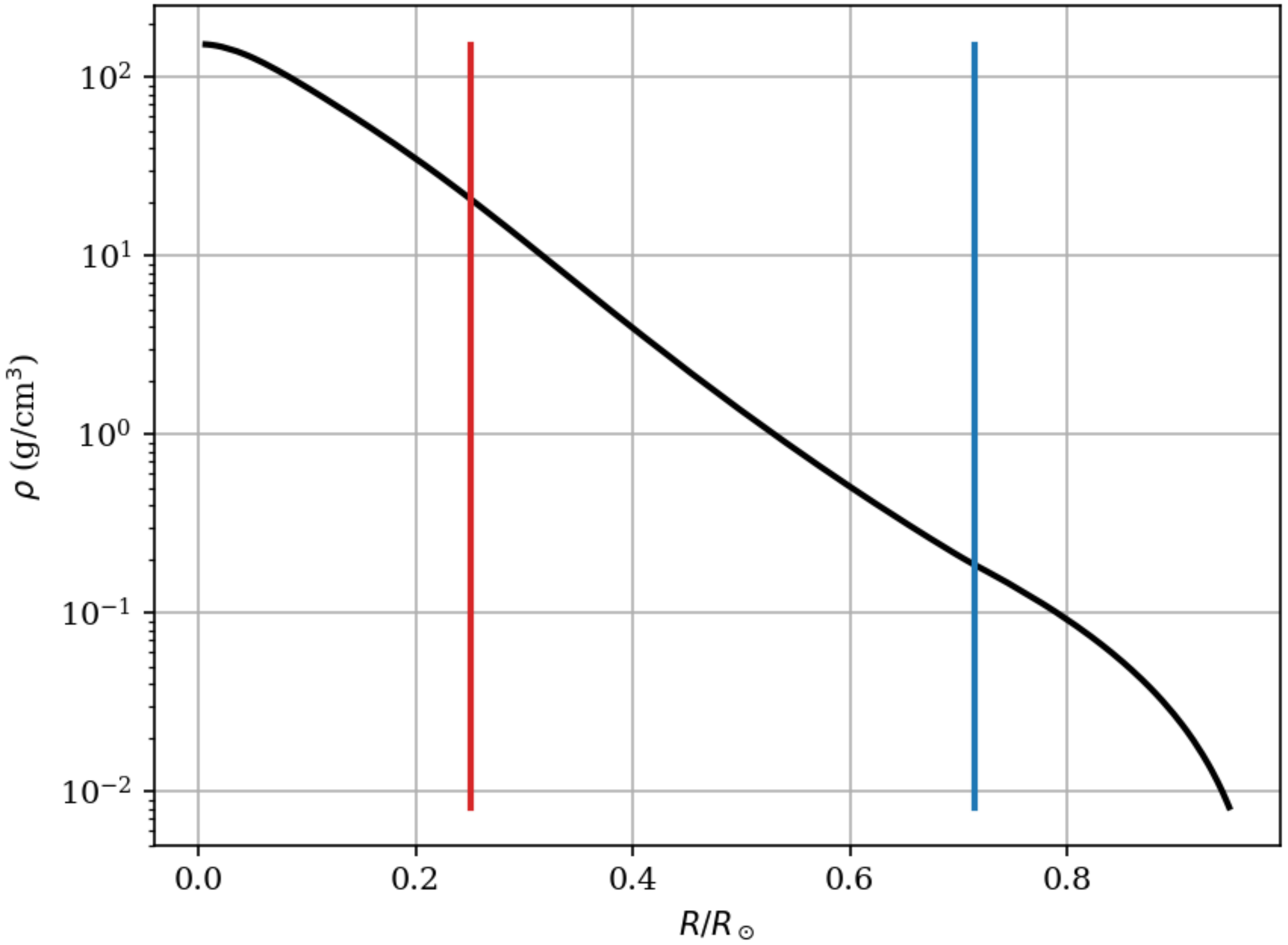
Standard Solar Model

- Very dense in core!
  - This is key for fusion…
- Convective zone is *fluffy!*
  - Water is 1 g/cm³
  - Osmium: 22 g/cm³



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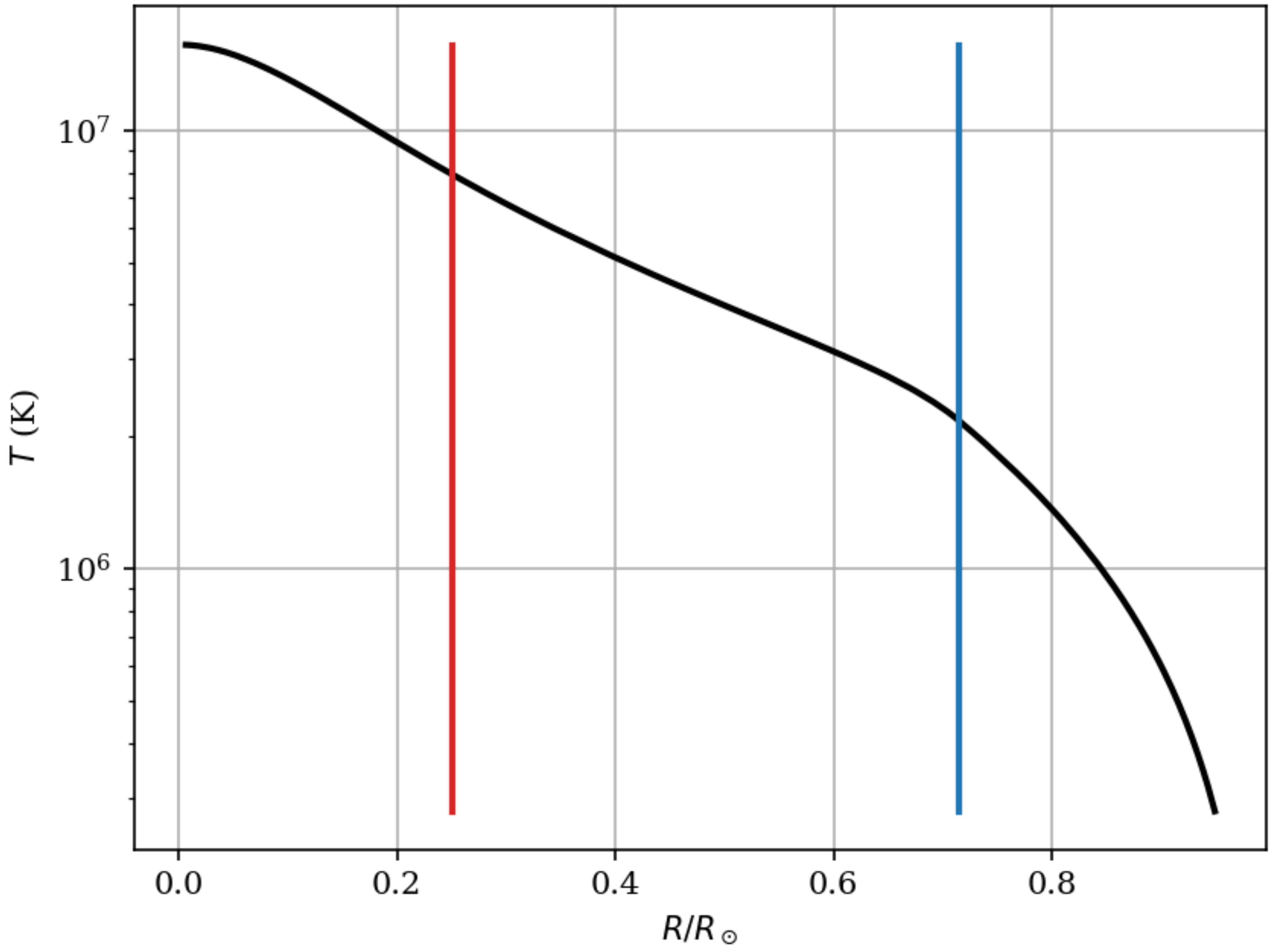
Standard Solar Model

- The inner core is VERY hot
  - Also key for fusion! Need very high velocities
- Radiative zone quite hot still, energy transportation primarily via radiation (hence the name)
- Convection zone, temp drops FAST



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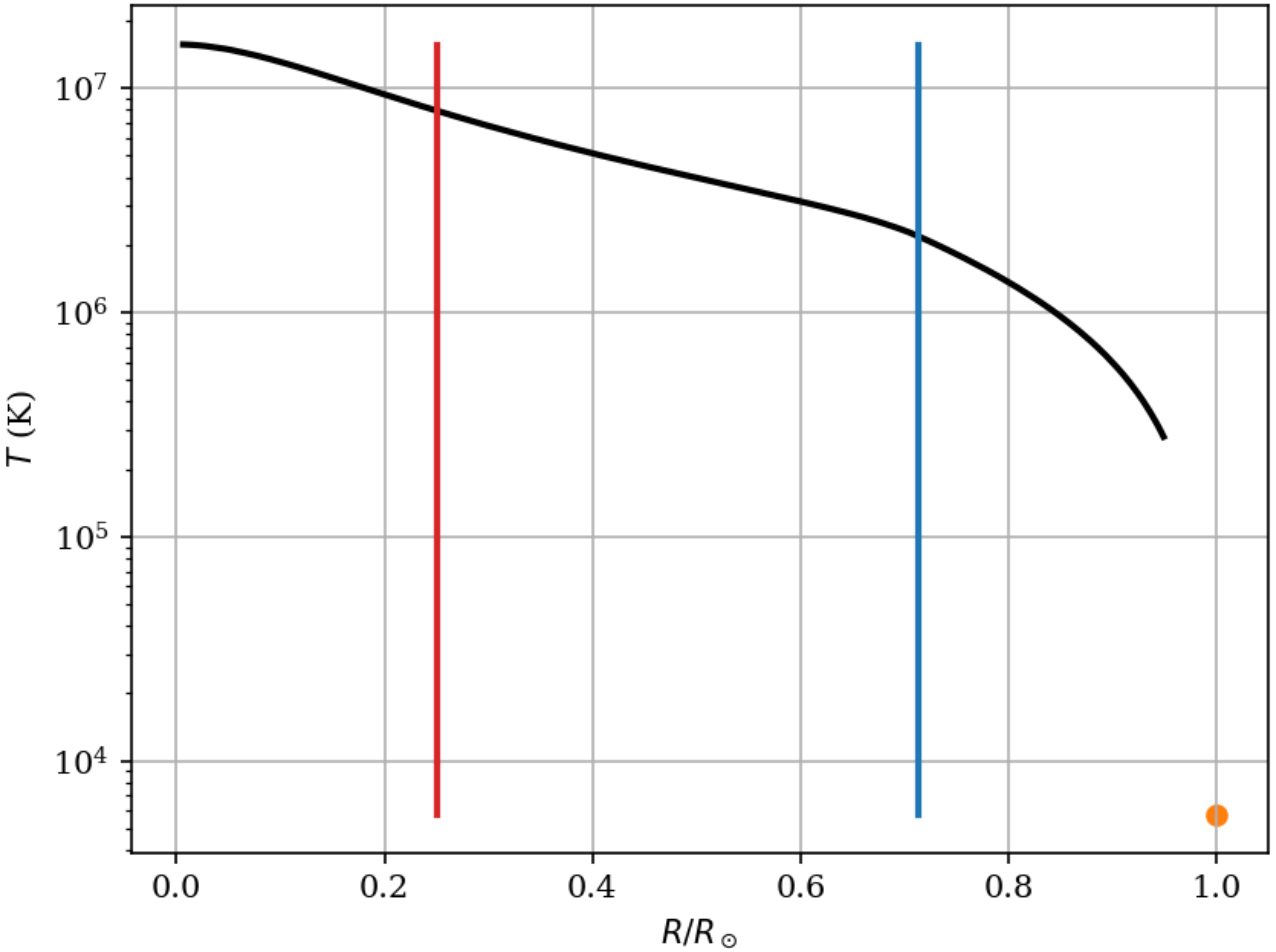
Standard Solar Model

- The inner core is VERY hot
  - Also key for fusion! Need very high velocities
- Radiative zone quite hot still, energy transportation primarily via radiation (hence the name)
- Convection zone, temp drops FAST
  - “Surface” temp of 5770 K



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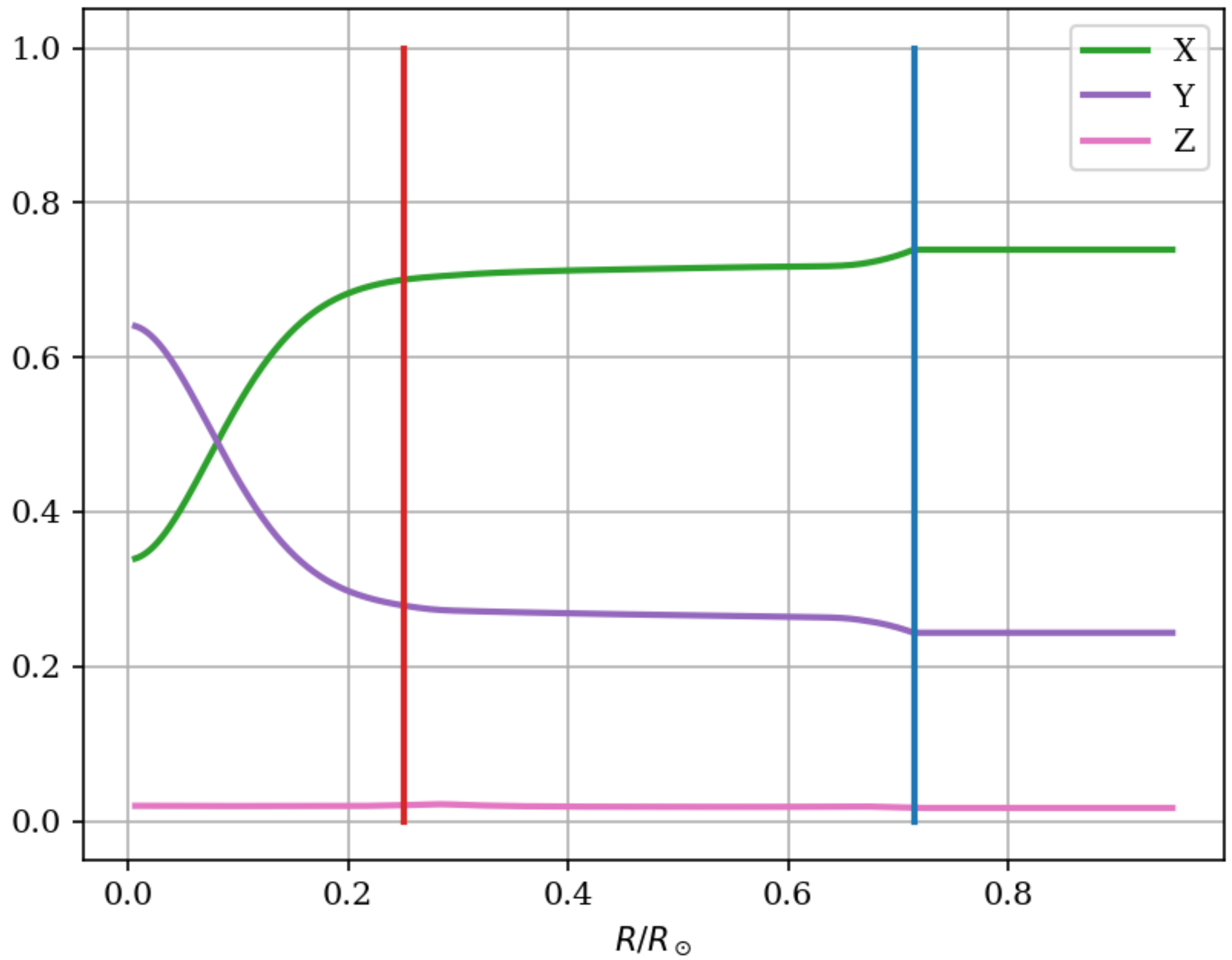
Standard Solar Model

- Mass fraction of H, He, and metals (X, Y, Z)
- Hydrogen is depleted in the core, helium is generated (fusion!)
- Fusion is strongest in deepest part of the core!



- Weak mixing of H, He through radiative zone
- TONS of mixing in convection zone

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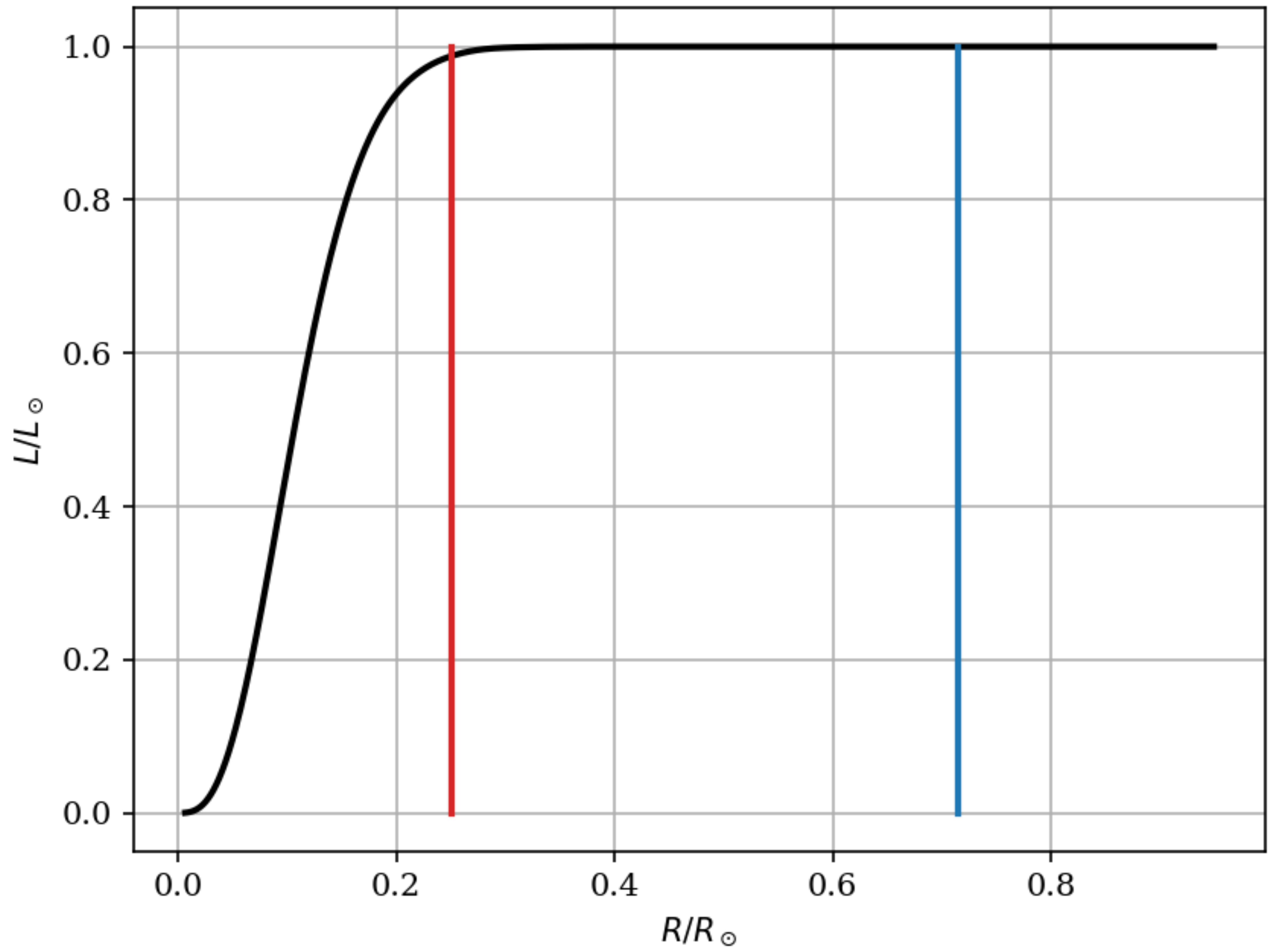
Standard Solar Model

- Luminosity comes from the core (fusion!)
- Fusion is extremely temperature dependent, strongest in deepest part of the core!
  - 50% of luminosity in just 10% of the radius
- ~no more luminosity generated in radiative or convective zones… star must maintain hydrostatic equilibrium, radiate energy to the surface!



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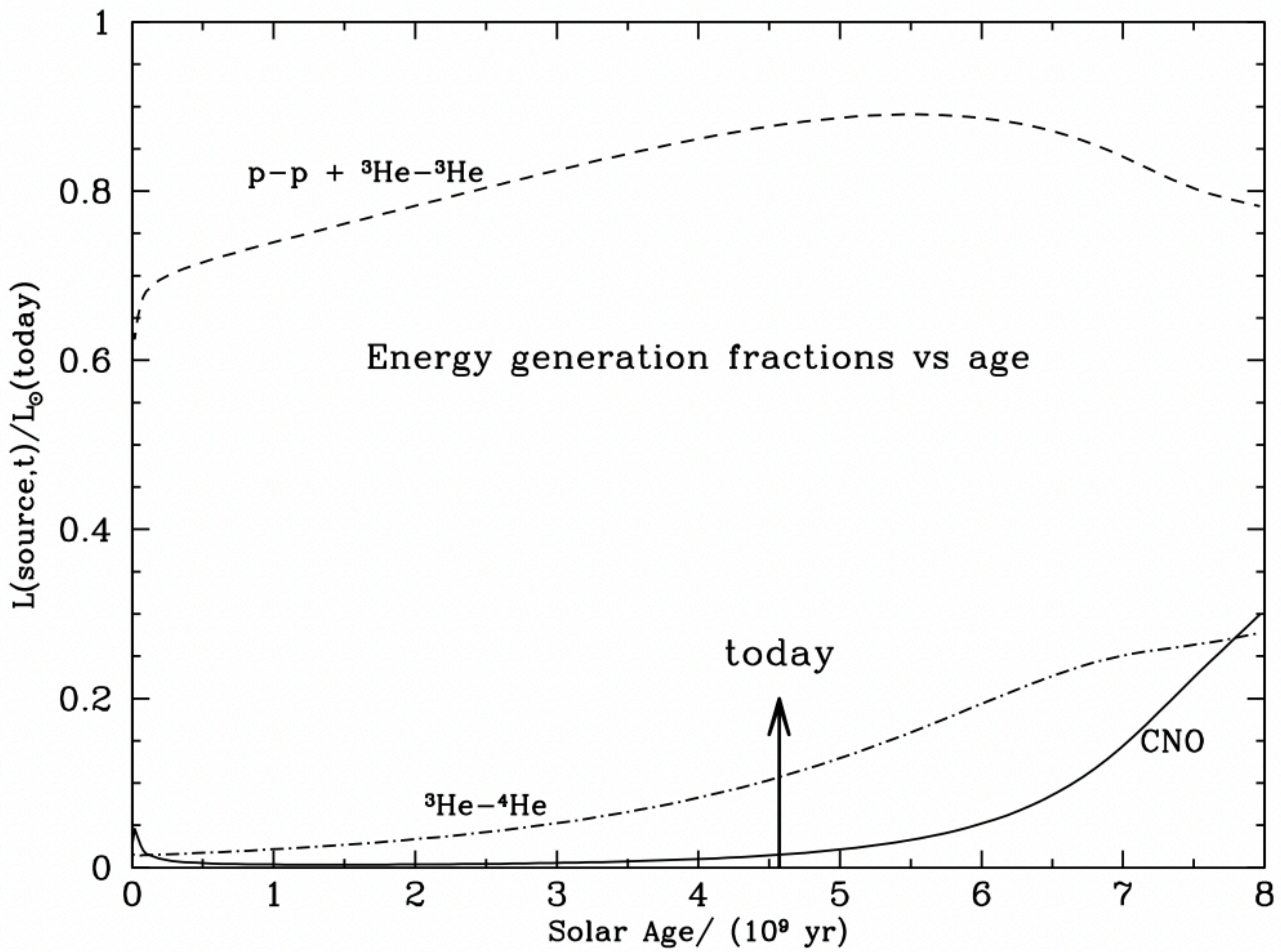
Standard Solar Model

- Fusion in the core, many branches not just the “p-p chain” for the Sun (4 H -> 1 He)
  - 2 branches of p-p chain dominate, contribute to the observed neutrinos (rates & energies) we detect on Earth!
  - CNO cycle (typically for massive stars) produces ~1.5% of the luminosity currently
    - Predicted in 1930’s, not confirmed for the Sun until 2020 via neutrino detectors!



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Stellar Timescales

- Let’s look at some general timescales at work, useful for estimating conditions of stars
  - Also very useful in other areas of astrophysics…
    (these are some of my favorite “order-of-mag” tools!)

Some of this is in BOB Ch 10
Some nice overviews are here:
https://www.astro.utu.fi/~cflynn/Stars/l3.html
https://www.astro.princeton.edu/~gk/A403/timescales.pdf

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Free Fall Timescale

- How long would a point mass take to fall from the surface due to gravity? (ignoring the actual star would be in the way)
- Or, how long does a body collapse under its own self-gravity (this is important in galactic dynamics, and studying gas cloud collapse)
- Point mass experiences acceleration $\frac{d^2r}{dt^2} = -\frac{GM}{r^2}$
- Integrate this, get a free-fall timescale of:

$$t_{ff} = \frac{1}{2}\sqrt{\frac{R^3}{GM}}$$

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Free Fall Timescale

$$t_{ff} = \frac{1}{2}\sqrt{\frac{R^3}{GM}}$$

- For the Sun, this is ~27min… wow!
  Gravity works fast
- Note $M/R^3 = \rho$, free-fall timescale depends on mean density

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Kelvin-Helmholtz Timescale

- AKA the "Thermal Timescale"
- t_{th} = total kinetic energy / rate of energy loss

only *slightly* better derivation of this in BOB, Ch 10.3

- This can be roughly written as:

  $E = \dfrac{GM^2}{R}$, and the energy loss is simple the luminosity. Thus:

- $t_{KH} \approx \dfrac{GM^2}{RL}$

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Kelvin-Helmholtz Timescale

$$t_{KH} \approx \frac{GM^2}{RL}$$

- For the Sun this is ~30 Myr… a long time, but not NEARLY long enough!
- This is the timescale a body would radiate all its energy away, clearly not the deciding factor in the lifetime of the Sun, but important for cooling timescales of dense gas (e.g. giant planets!)
- +100yrs ago, this was the best constraint on the age of the Sun… geology starts to put stronger constraints on age!

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Nuclear Timescale

- AKA the “Einstein Timescale”
- Using some *very hand-wavey* logic (classic astronomers…), and everybody’s favorite $E = mc^2$
- $t_{nuc} = \dfrac{E_{nuc}}{L}$
- If the *entire* Sun were to undergo fusion:

$$t_{nuc} = \frac{M_\odot c^2}{L_\odot} \approx 10^{13} \text{ yrs.}$$

Thankfully this doesn’t happen…

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Nuclear Timescale

$$t_{nuc} = \frac{E_{nuc}}{L}$$

- Instead, go back to simplest view of p-p chain fusion: 4 H -> He + energy
- The difference in mass between 4 H and He is 0.7%, that's the amount released in energy
- Not all H can be fused into He, reactions very sensitive to Temp and density, eventually not enough protons zipping around to keep it up (end of main seq!)
- A slightly better estimate is ~10% of the Sun's mass is available for fusion (i.e. in the inner core) $E_{nuc} = 10\% \times 0.7\% \times M_{\odot} c^2$
- $t_{nuc} = \frac{E_{nuc}}{L} \sim 10^{10}$ years

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Next Time

- Central pressure
- Stellar Structure Equations
- Polytropes
    - The “Lane-Emden Equation”

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