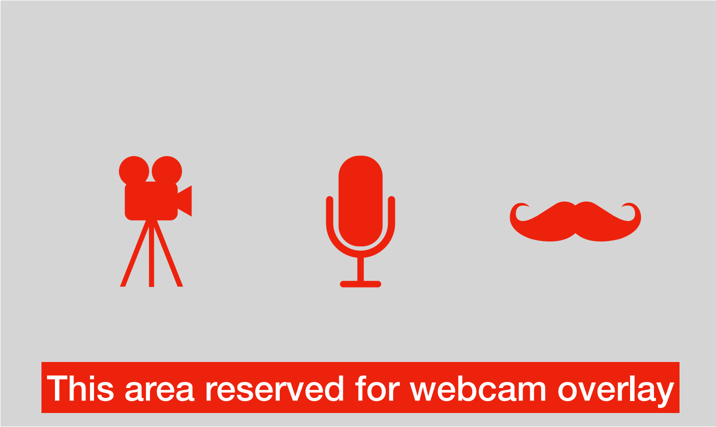
ASTR 421

Stellar Observations and Theory

Lecture 07
Radiative Transfer

Prof. James Davenport (UW)



Starting section on Radiative Transfer

- "Radiative Transfer" is simply the study of how energy (photons) are transferred to/from a medium of gas
- We've already been working on many of the components!
  - Emission (lines, continuous, etc)
  - Absorption
    - Opacity
    - Optical depth

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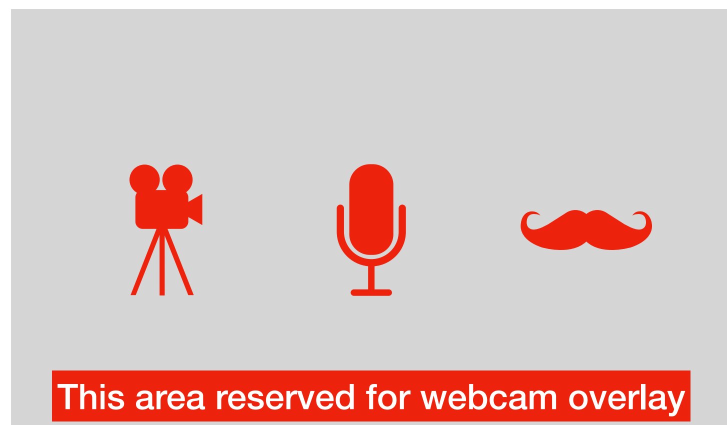
Remember Opacity & Optical Depth

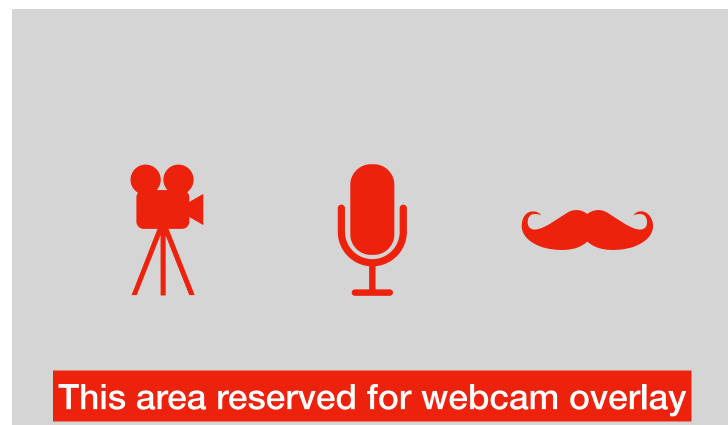
- (Lecture 5)

- We defined the change of intensity through a medium due to all forms of opacity as: $dI_\lambda = -\kappa_\lambda \, \rho \, I_\lambda ds$
  - Where κ is our friend, the opacity coefficient (or sometimes called the absorption coefficient)
  - We considered regimes where this absorption was dominant (i.e. optically thick)



How about emission?

- Many of the sources of scattering and absorbing we discussed have *emission* analogs
  - Bound-bound (lines), bound-free (radiative recombination)
  - “Inverse” scattering (or braking) processes
  - Thermal (blackbody) continuum

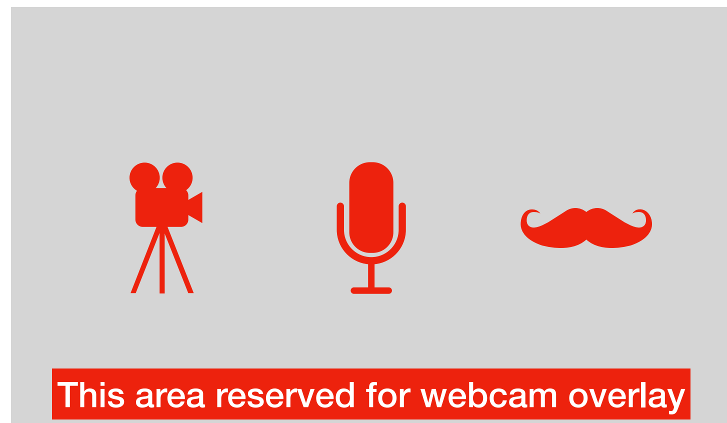


How about emission?

Note: j is sometimes written as ε

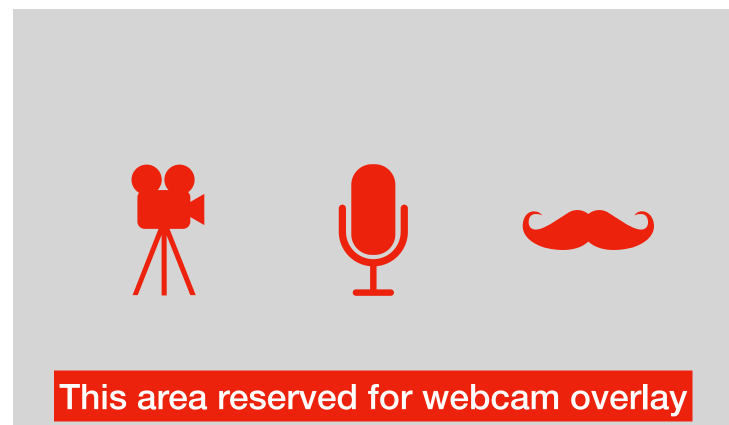
- We can then define an *emission coefficient* (j): $dI_\lambda = j_\lambda \, \rho \, ds$
- As before, can integrate this to demonstrate pure emission:
  - $I_\lambda = e^{\,j_\lambda \, \rho \, s}$

  - We don't usually write it this way, its not physical



How about emission?

- We can then define an *emission coefficient* (j): $dI_\lambda = j_\lambda \, \rho \, ds$
- Compare/contrast this with the definition for absorption: $dI_\lambda = - \kappa_\lambda \, \rho \, I_\lambda ds$
  - There is no (-) sign
  - There is no initial intensity I_λ term, since you can have emission w/o absorption, of course.



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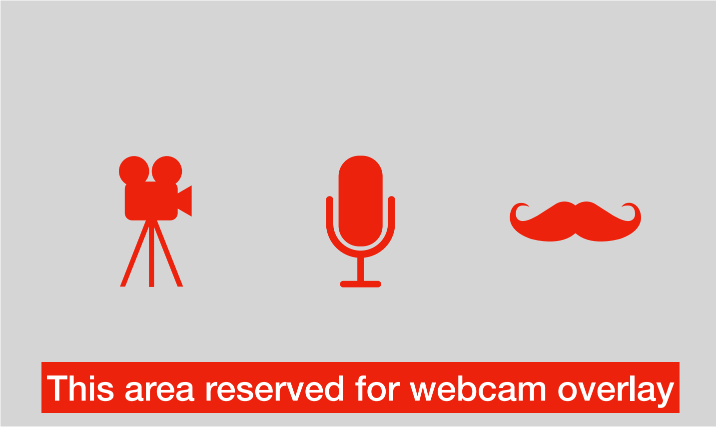
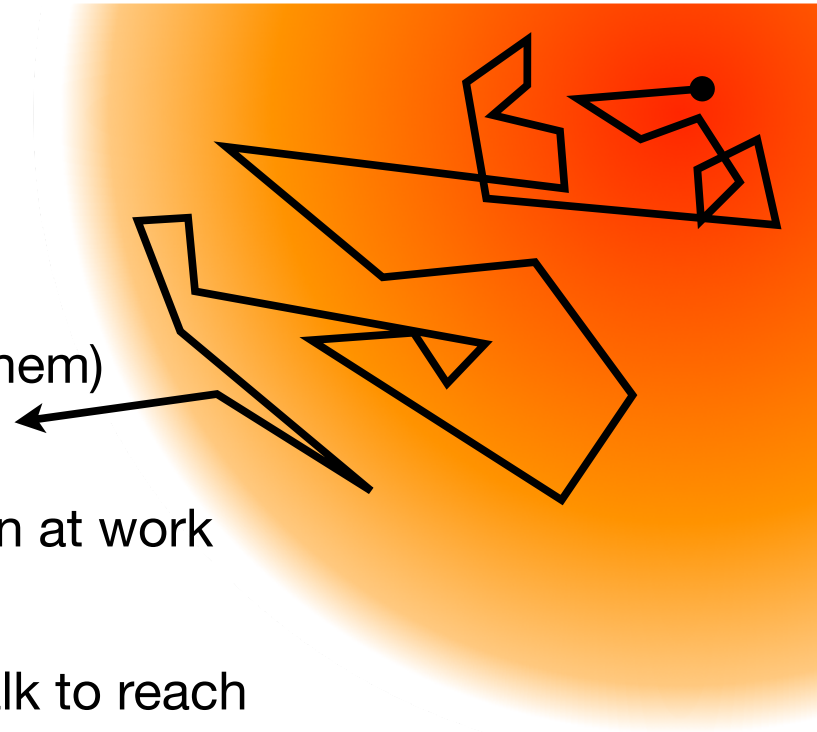
In most atmospheres there is always both emission and absorption going on!

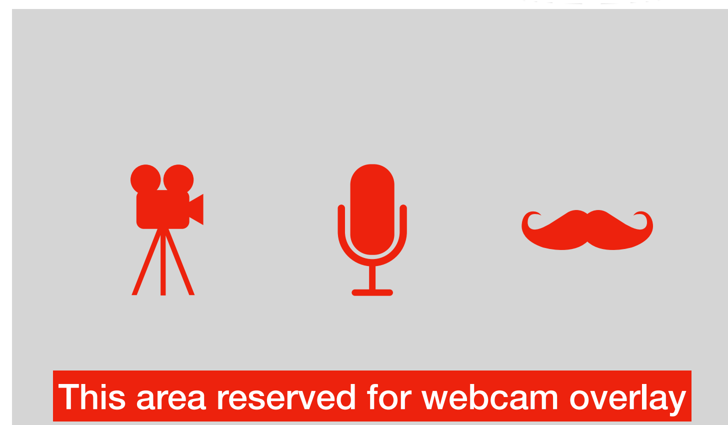


How do photons reach us?



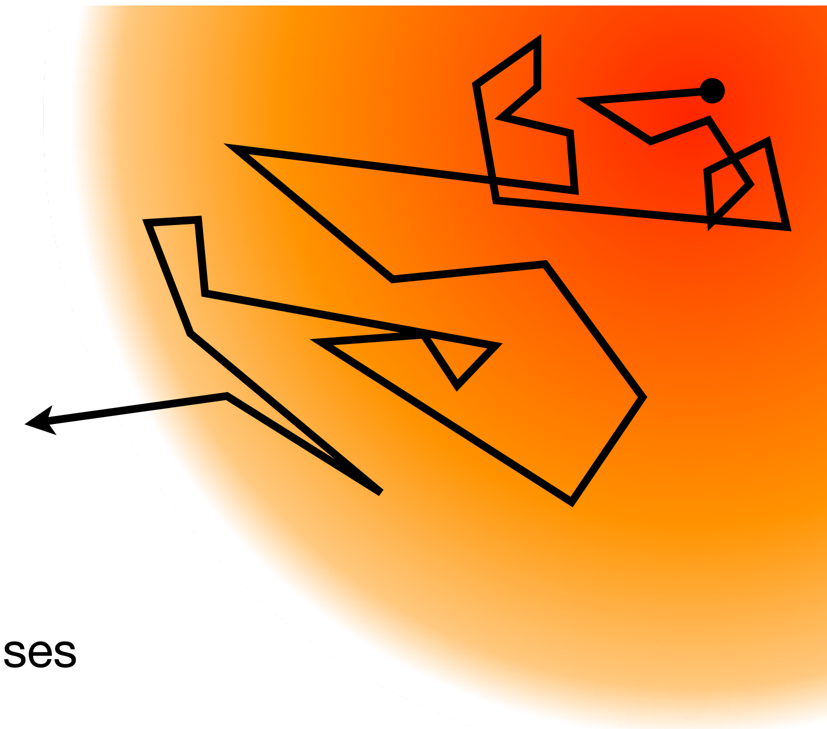
- Remember it takes a LONG time for photons to wander from the core (where fusion is *creating* them) to the "surface" (i.e. where they finally escape)
- This **is** the processes of absorption and emission at work (a "random walk", as the book defines it)
- The book defines the number of steps in this walk to reach the surface as: $N = \tau_\lambda^2$
    - Total optical depth of the sun is $\tau \sim 10^{11}$ so… that's a *lot* of steps.

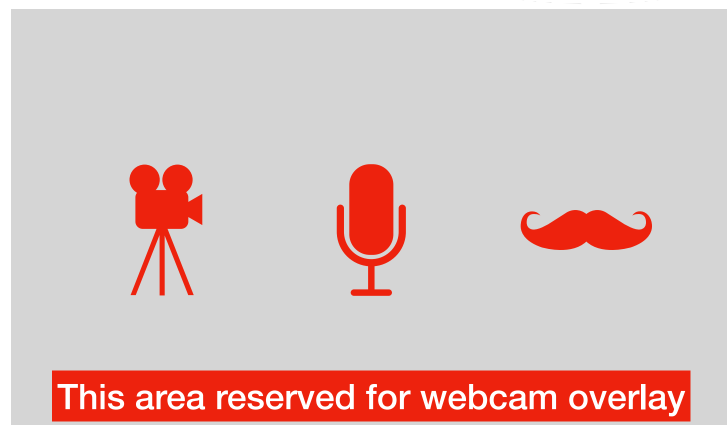
Estimate from: https://www.maplesoft.com/applications/view.aspx?SID=154320&view=html



How do photons reach us?

- The star is (generally) in *equilibrium*
  - All photons generated in core must escape
  - At any given thin layer in the star, very little "flow" of photons upward (toward surface)
- There *are* gradients in temp, density, pressure… causes a slight "breeze" of photons upwards
- If photon "energy transport" is *too* low, need some other way to move energy out to the surface to keep in equilibrium…
  - We'll get back to this when we talk about convection & stellar structure…





Emission & Absorption

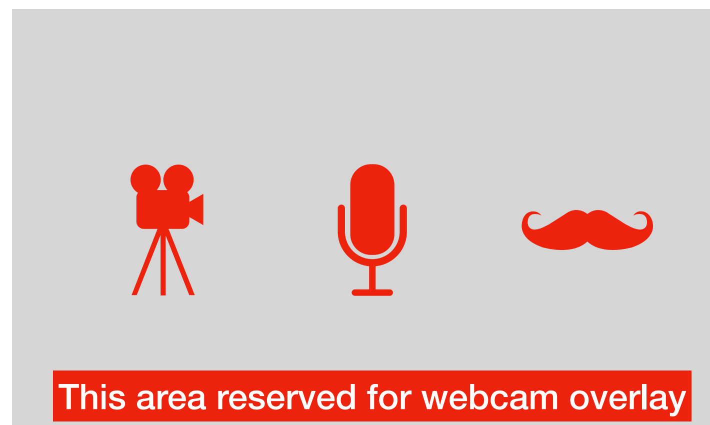
- **Both emission and absorption are happening within a volume of gas!**

$$dI_\lambda = j_\lambda \, \rho \, ds \qquad\qquad dI_\lambda = -\,\kappa_\lambda \, \rho \, I_\lambda ds$$

- Thus it stands to reason you can combine these two processes:

$$dI_\lambda = (-\kappa_\lambda \, \rho \, I_\lambda \, ds) + (j_\lambda \, \rho \, ds)$$

- This is one (simple) way to express the so-called “transfer equation”

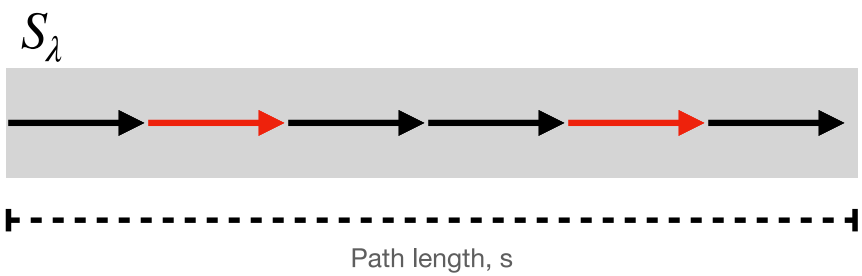


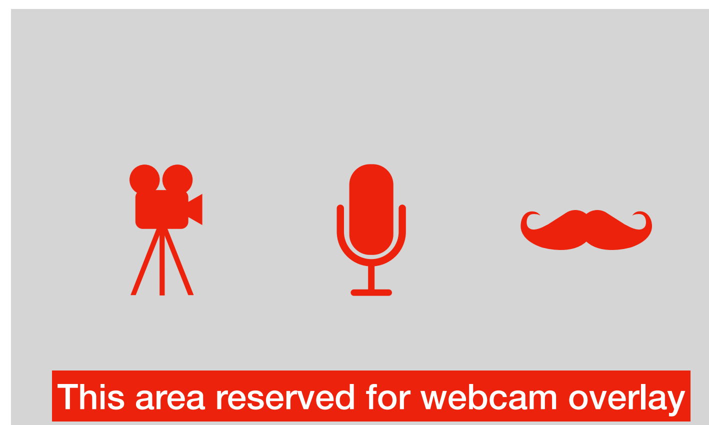
Source Function

- The ratio of the emission to absorption coefficients:

$$S_\lambda \equiv \frac{j_\lambda}{\kappa_\lambda}$$

- Describes how photons are emitted & removed by a gas





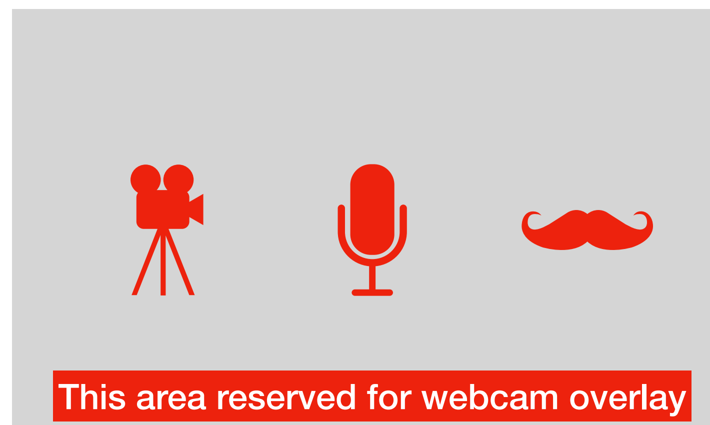
Source Function

$$S_\lambda \equiv \frac{j_\lambda}{\kappa_\lambda}$$

- Can be thought of as a "specific intensity" of how much light is emitted at a given point
- **Interestingly, has the same units as intensity!**

$$dI_\lambda = j_\lambda \, \rho \, ds \qquad dI_\lambda = -\kappa_\lambda \, \rho \, I_\lambda ds$$

  - Can infer this (via dimensional analysis) since the I_λ term is missing from the emission side
  - $[\kappa] = \mathrm{cm}^2/\mathrm{g}$
  - $[j] = [I\kappa]$
    - $[j/\kappa] = [I]$



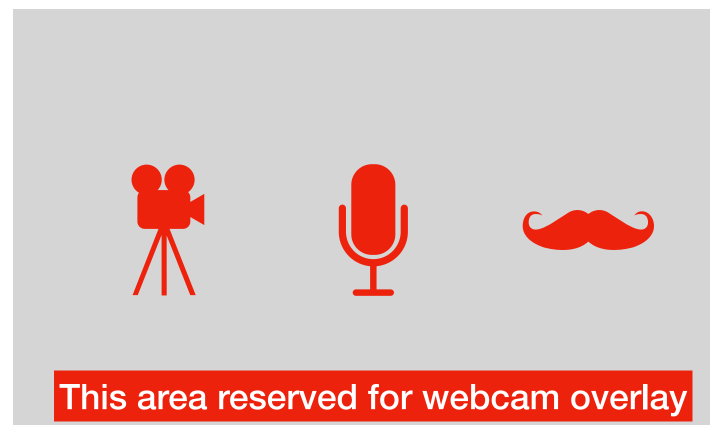
Source Function

$$S_\lambda \equiv \frac{j_\lambda}{\kappa_\lambda}$$

- This is very helpful, because we're VERY familiar with a certain formula for the intensity of light in special cases:

$$B_\lambda(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

- i.e. for a dense (thick) material in Thermodynamic Equilibrium, $S_\lambda = B_\lambda$



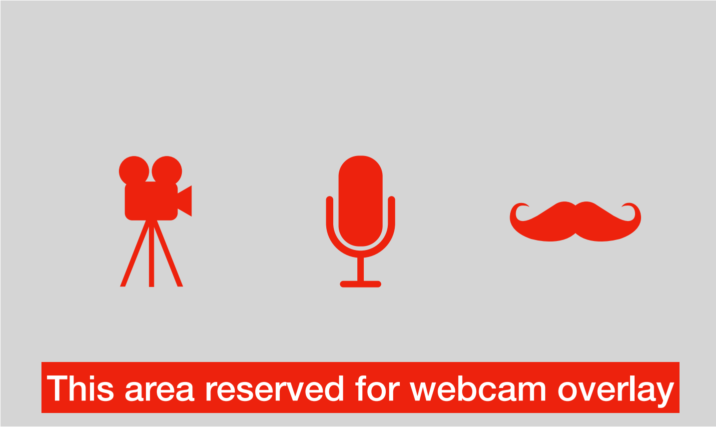
Radiative Transfer Equation

*in differential form

- We have combined emission & absorption in our medium
$$dI_\lambda = (-\kappa_\lambda \, \rho \, I_\lambda \, ds) + (j_\lambda \, \rho \, ds)$$

- If we divide both sides by $-\kappa_\lambda \, \rho \, ds$, we get:
$$-\frac{1}{\kappa_\lambda \rho} \frac{dI_\lambda}{ds} = I_\lambda - S_\lambda$$
$$S_\lambda \equiv \frac{j_\lambda}{\kappa_\lambda}$$

- We can also bring our friend optical depth back in…
$$\frac{dI_\lambda}{d\tau_\lambda} = I_\lambda - S_\lambda$$



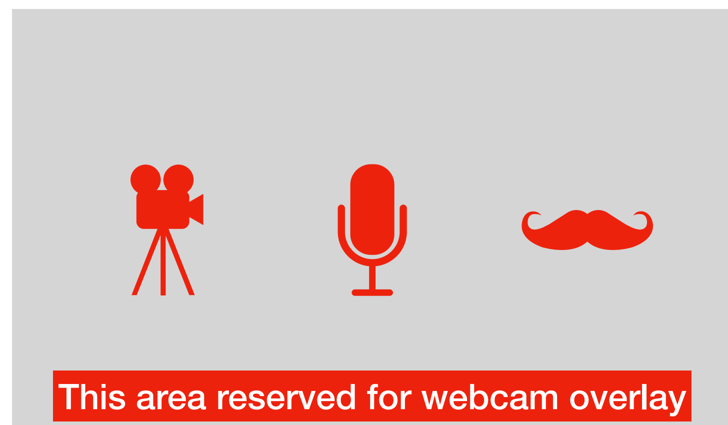
Radiative Transfer Equation

- This can be “solved” to find the intensity of light as a function of path length through a medium:

$$\frac{dI_\lambda}{d\tau_\lambda} = I_\lambda - S_\lambda$$

$$I_\lambda \;=\; I_{\lambda,0}\, e^{-\tau_\lambda} + S_\lambda\,(1 - e^{-\tau_\lambda})$$

- This is a solution only in a “linear” case
- IRL you need to consider 3D radiative transfer…

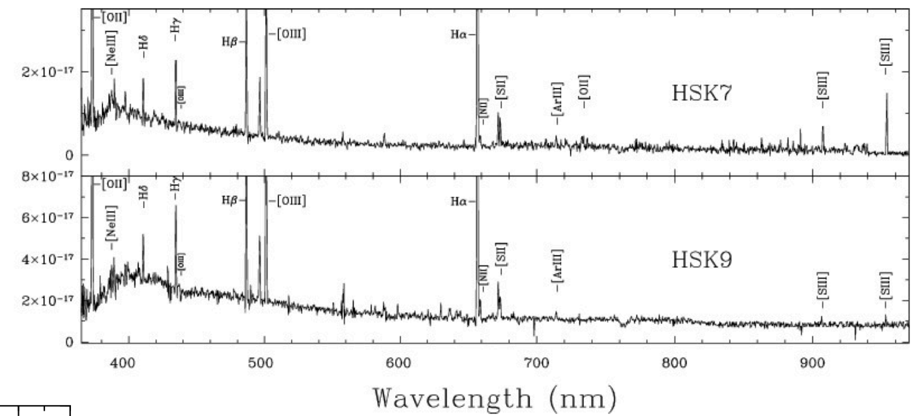


Returning to Emission

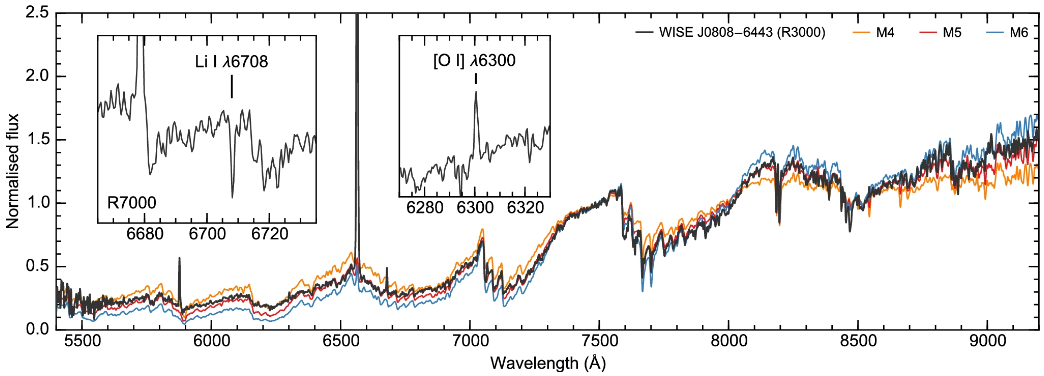
- Lots of focus on absorption & thermal emission (B)
- But let's not forget line emission too!
    - Obv. need gas to be hot/heated
    - How do these lines actually occur?

Planetary Nebulae

adsabs.harvard.edu/abs/2005A%26A...443..115M



M Dwarf



https://ui.adsabs.harvard.edu/abs/2018MNRAS.476.3290M/abstract

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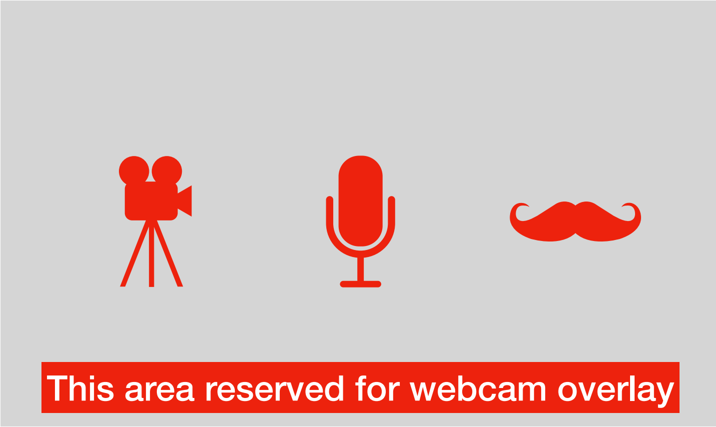
Returning to Emission

- For simplicity, assume there is no radiation entering some pocket of gas (i.e. neglect absorption)
  $I_\lambda = \cancel{I_{\lambda,0}\, e^{-\tau_\lambda}} + S_\lambda \,(1 - e^{-\tau_\lambda})$

- Assume the cloud is optically thin (τ is very small). Use Taylor series to expand: $e^{-\tau_\lambda} \approx 1 - \tau_\lambda + \ldots$

- So the intensity then becomes:
  $I_\lambda \approx \tau_\lambda S_\lambda$

- i.e. the emission intensity will be strong at λ's with large τ (but… it's optically thin, so…)



This example is from:
http://www.physics.usyd.edu.au/~helenj/SeniorAstro

Returning to Emission

- Recall *again* our friendly definition of optical depth: $d\tau_\lambda = -\kappa_\lambda \rho \, ds$
- τ is larger where κ is higher - i.e. primarily emission lines! (consider again all the sources of emission, lines often dominate)
- So finally: $I_\lambda \approx \tau_\lambda S_\lambda \propto \kappa_\lambda B_\lambda$
- In an optically thin region, very strong emission lines can occur
- This actually an important *cooling* mechanism for low density gas…
- Why PNe have such strong emission lines!

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Returning to Emission

- What about emission lines from optically thick sources?
- $I_\lambda \; = \; I_{\lambda,0}\, e^{-\tau_\lambda} \; + B_\lambda \; (1 - e^{-\tau_\lambda})$
  can't neglect absorption…
- $I_{\lambda,0} = B_\lambda(T_{in})$, and the source function (right) side is $B_\lambda(T_{out})$ (assuming LTE)
- Again, fun w/ Taylor Series…
  $I_\lambda(\tau_\lambda) \approx B_\lambda(T_{in}) + \tau_\lambda[B_\lambda(T_{out}) - B_\lambda(T_{in})]$

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Returning to Emission

- $I_\lambda(\tau_\lambda) \approx B_\lambda(T_{in}) + \tau_\lambda[B_\lambda(T_{out}) - B_\lambda(T_{in})]$
- So in "normal" scenario, $T_{out} < T_{in}$, resulting intensity is lower i.e. absorption lines
- BUT, if $T_{out} > T_{in}$, you see higher intensity (i.e. emission)
    - This happens e.g. in the solar Transition/Corona regions, where density decreases but temperature increases
        - Lines are weak, but present
    - Also in the chromosphere, right above surface

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Returning to Emission

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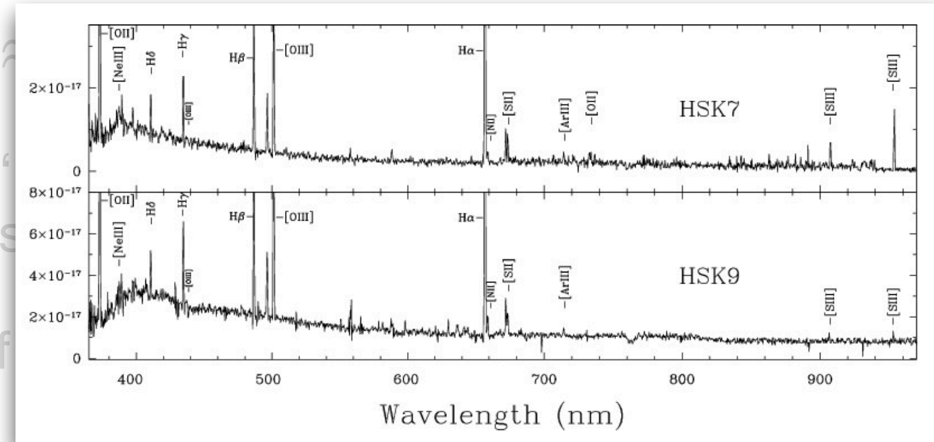

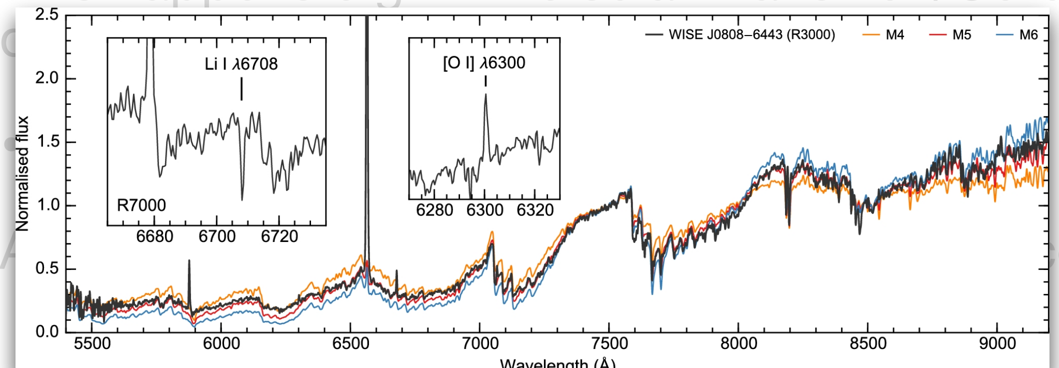

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Next time:

- Starting putting pieces together to understand line profiles more!
- HW 3 is now posted